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QCD and the Tetron Model

Bodo Lampe

II. Institut für theoretische Physik der Universität Hamburg
Luruper Chaussee 149, 22761 Hamburg, Germany

Abstract

We present a derivation of Quantum Chromodynamics (QCD) and color confinement emerging from a 7+1-dimensional microscopic theory. In this theory quarks are excitations of fundamental 7+1D fermions (the Tetrons) which are condensed into a 3+1D monolayer vacuum structure, and gluons arise as collective topological link excitations. The $SU(3)_C$ gauge symmetry is not postulated ad hoc, but emerges dynamically as an effective macroscopic description of the topological transport on a discrete 3-dimensional 'monolayer'. Furthermore, we show that color confinement arises naturally via a modified dual Meissner effect. Unlike standard phenomenological models that rely on the explicit condensation of magnetic monopoles, the confining medium in this framework is dynamically generated by the geometric frustration and chiral topology of the Tetron lattice. The discrete isomagnetic nature of the vacuum forces emergent color-electric flux lines into one-dimensional tubes, yielding a rigorous microscopic origin for the linear confining potential.

1 Introduction

The formulation of Quantum Chromodynamics (QCD) as the exact gauge theory of strong interactions has achieved remarkable success in the high-energy regime due to asymptotic freedom. However, at low energies color confinement has not been proven for the contiuum version of QCD, and is one of the most profound unresolved problems in theoretical physics. While the dual superconductor picture[1] proposed in the 1970s provides a compelling macroscopic analogy by attributing confinement to a dual Meissner effect via monopole condensation, the microscopic origin of this superconducting vacuum medium has historically remained elusive. Standard approaches often require the introduction of ad hoc scalar fields or topological defects without a fundamental geometric derivation[2, 3]. In this paper, we resolve this structural deficit by deriving both the $SU(3)_C$ gauge symmetry and a modified Meissner effect directly from the geometric first principles of the Tetron model[4].

The Tetron framework postulates a fundamental 7+1-dimensional spacetime governed by $SO(7,1)$ symmetry, wherein Standard Model fermions emerge as composite states of elementary Dirac fermions (Tetrons). Spontaneous symmetry breaking reduces this high-dimensional space to a 3+1-dimensional macroscopic monolayer, defined by a rigid, discrete lattice of tetrahedral 'all-out' isospin configurations. This chiral vacuum topology has previously been shown to successfully dictate the electroweak flavor hierarchy, giving a microscopic interpretation to the Higgs mechanism and generating quark and lepton masses[4].

Here, we extend the macroscopic consequences to include the strong interaction. We identify the origin of the $SU(3)_C$ color degrees of freedom with the internal permutation symmetries of Tetrons inside the isomagnetic unit cells. At distances much larger than the lattice constant (the Plamck length), the discrete phase shifts accumulated by Tetrons propagating through the tetrahedral boundary conditions manifest as a continuous $SU(3)_C$ gauge theory.

Crucially, the translation from the discrete bulk to the continuous effective field theory reveals that the isomagnetic tetrahedral all-out vacuum inherently acts as a topological color-superconductor. The modified dual Meissner effect derived in this work is driven not by magnetic monopoles or scalar Higgs-like condensates, but by isomagnetic couplings in the 4 extra dimensions. The bulk lattice strictly expels emergent color-electric fields, i.e. the geometric incompatibility between the lattice rigidity and the color flux forces the gauge fields into flux tubes between two Tetron triplet excitations (quarks).

By detailing the exact transition from the microscopic lattice Hamiltonian to the macroscopic QCD Lagrangian, we provide a unified geometric mechanism in which the strong force and color confinement are inevitable structural consequences of the chiral Tetron vacuum.

2 Details of the Model

Within the 7+1 dimensional spacetime there are fermions transforming under the fundamental 16D representation $8_L + 8_R$, where 8_L and 8_R are the two complex spinorial representations[5] of $SO(7,1)$, one with left handed and the other with right handed chirality. When decomposing $SO(7,1)$ into the Lorentz symmetry $SO(3,1)$ of a 'base' Minkowski spacetime and an 'internal' $SO(4)$, describing the symmetry group of the 4 extra dimensions, the representations decompose as

$$\begin{aligned} SO(7,1) &\rightarrow SO(3,1) \times SU(2)_L \times SU(2)_R \\ 8_L &\rightarrow ((1,2),(2_L,1)) + ((2,1),(1,2_R)) \\ 8_R &\rightarrow ((1,2),(1,2_R)) + ((2,1),(2_L,1)) \\ 8_L \oplus 8_R &\rightarrow ((1,2)+(2,1),(1,2_R)+(2_L,1)) \end{aligned} \tag{1}$$

where the covering group $SU(2)_L \times SU(2)_R$ of $SO(4)$ has been introduced. $(2,1)$ and $(1,2)$ denote left and right handed Weyl spinor representations of $SO(3,1)$ while the left and right handed $SO(4)$ spinors are denoted by $(2_L,1)$ and $(1,2_R)$. Note, the chiralities of all the doublets appearing in (1) are intertwined because they follow from the chiralities of the parent representations 8_L and 8_R.

These fermions are called Tetrons Ψ, and it is assumed that the observed quarks and leptons arise as vibrations $\delta\vec{Q}$ ('iso-magnons') of isospin vectors in the 4 extra dimensions:

$$\vec{Q}_L = \frac{1}{4}\Psi^\dagger(1-\Gamma_5)\vec{\tau}\Psi = \frac{1}{2}\Psi_L^\dagger\vec{\tau}\Psi_L \qquad \vec{Q}_R = \frac{1}{4}\Psi^\dagger(1+\Gamma_5)\vec{\tau}\Psi = \frac{1}{2}\Psi_R^\dagger\vec{\tau}\Psi_R \tag{2}$$

In order to obtain the complete vibrational degrees of freedom, the decompositon of $8_L \oplus 8_R$ in the last line of (1) should be considered. The corresponding Tetron field represents a Dirac field $(1,2)+(2,1)$ in the $SO(3,1)$ base part, with 2 $SO(4)$ doublets $\Psi_L=(U_L,D_L)$ and $\Psi_R=(U_R,D_R)$ on top.

Note that the indices L and R refer to the chiralities in the 4 extra dimensions. As well known, any rotation in $SO(4)$ can be described by a pair of 3D angular momentum vectors $\vec{Q}_L$ and $\vec{Q}_R$, corresponding to 2 simultaneous rotations of 2 planes in R^4. (The notion of a rotation axis is not meaningful in 4D.) In the special case $\vec{Q}_L=\vec{Q}_R$ this reduces to one rotation in just one plane.

The ground state of the system has been discussed at length in previous publications[4, 6]. According to this, at Planck scale distances our universe is a discrete structure consisting of aligned isospin tetrahedrons extending into the 4 extra dimensions. More precisely, the ground state consists of unit cells extending into the extra dimensions where the isospin system of the Tetrons forms an all-out configuration, with 2 aligned tetrahedrons T_L and T_R each formed by 4 isospin vectors Q_L and Q_R, respectively, and living in the two 3D isospin spaces mentioned above. This means, at each tetrahedral corner $i=1-4$, vectors $Q_{L,i}$ und $Q_{R,i}$ are parallel and point in the tetrahedral easy axis direction.

Due to the pseudovector property of the isospin vectors, the point group of the tetrahedral unit cell is the Shubnikov group

$$G_4 := A_4 + CPT(S_4 - A_4) \tag{3}$$

where $A_4(S_4)$ is the (full) tetrahedral symmetry group. G_4 corresponds to an unbroken symmetry which holds down to the lowest energies. It has only 1- and 3-dimensional representations and describes all 24 SM quarks and leptons after the SSB.

With respect to 3+1D physical space the unit cells appear pointlike, and distributed amorphously, with the average distance between the points being the Planck length.

While gravity can be attributed to the elasticity of the coordinate bonds[7], the phenomena of particle physics arise from the interactions between the isospin vectors forming the tetrahedrons.

Isospins Q_L and Q_R (2) form the generators of the group $SU(2)_L \times SU(2)_R$, which acts on the 4 extra dimensions. This 4D isospin space on which the tetrahedral unit cells are hosted is a subspace of the higher-dimensional 7D manifold. Adjacent unit cells are distributed exclusively in the dimensions orthogonal to the extra $\mathbb{R}^4$. Consequently, the all-out tetrahedral array forms a topological monolayer within the 7D bulk. This monolayer constitutes our physical universe; in the continuum limit, as the spatial lattice (but not the isospin structure) becomes unresolvable, it maps onto the familiar 3+1 dimensional Minkowski spacetime, cf. the details given in the next section.

Clearly, the expansion into a relativistic spacetime continuum introduces a significant constraint: the tetrahedral isospin symmetry is strictly defined only in the rest frame of the lattice. However, the quasiparticle excitations which make up for the ordinary matter and propagate along the monolayer exhibit different symmetry properties. Because these excitations are wave-like solutions within the 3+1 dimensional effective manifold (the monolayer), they manifest as Lorentz $SO(3,1)$ covariant objects. In this interpretation, while the fundamental vacuum is non-covariant and discrete, the observable physical sector – consisting of collective excitations – obeys the laws of special relativity. These particles 'glide' over the monolayer substrate, treating the underlying non-invariant isospin grid as a relativistic medium.

3 The Spatial Continuum Limit and the Recovery of Lorentz Invariance

In this work we are considering particle physics effects only, which amounts to switching off gravity (and the corresponding elasticity of the spatial bonds) by considering the continuum limit with respect to physical space, while keeping the discrete tetrahedral isospin configuration intact. Actual calculations should be done in the rest system of the tetrahedral configurations, which on cosmic scales amounts to the so-called CMB rest system[8]. The expanding universe thus is not the expansion of an empty space, but of the increasing distance between the tetrahedral cells, corresponding to the uniform expansion of the monolayer grid.

It is well known that the fundamental spacetime constants c, $\hbar$ and G can be used to define the Planck length, time and mass L_p, T_p and M_p which describe the basic properties of space[m], time[s] and matter[kg]

$$c = \frac{L_p}{T_p} \qquad \hbar = E_p T_p \qquad \kappa = \frac{L_p}{E_p} \tag{4}$$

where $E_p = M_p c^2$ is the Planck energy and $\kappa = G/c^4$ the Einstein constant.

Our monolayer is a 'Planck lattice' – more precisely a discrete amorphous structure – in the sense that we identify the average spacing between 2 unit cells as the Planck length L_p. Since we are not interested in gravitational but only in particle physics effects, we are allowed to take the limit $G \to 0$ while keeping c and h fixed. This removes the spatial elasticity, at the same time allowing to consider the continuum limit $L_p \to 0$ without giving up the discrete structure in isospin space.

According to (4) this limit implies $E_p \to \infty$ and corresponds to the observed large value of the Planck energy, which is interpreted as the binding energy of the spatial 'lattice', while the isomagnetic couplings – to be discussed later after Eq. (5) – with magnitude $O(E_F)$ are the much smaller interaction energies among the isospins.

In other words, this limit sends G to zero while simultaneously allowing to use the continuum limit for the spatial coordinates and considering a system of discrete isospin vectors forming aligned tetrahedrons in the ground state.

As $E_p \to \infty$, the medium (which may be considered a kind of 'Lorentz ether') becomes infinitely stiff against gravitational deformations. What remains are the internal degrees of freedom of the symmetry group $SU(2)_L \times SU(2)_R$. If we consider E_p as the gigantic binding energy of the lattice (10^{19} GeV) and E_F as the gentle interaction energy of the isospins (10^2 GeV), we find ourselves in a regime where the 3D lattice structure is rigid but disordered (amorphous), while the isospin degrees of freedom act upon it like an ordered and relatively smooth structure. It is a sort of nematic system, where 'molecules' consisting of 4 isomagnets are in an ordered/aligned phase but their positions in 3D physical spaceare are disordered, similar to a glassy liquid[9].

We are now in the position to recover Lorentz invariance as an emergent symmetry of the low-energy sector[10]. The preferred frame of the Planck-lattice exists, but it is hidden behind the energy barrier E_p. For the matter excitations (quarks and leptons), the geometry of the monolayer effectively behaves as a $3+1$ dimensional Minkowski space.

More in detail, the vacuum of the present theory is a structured 'isospin crystal', with a lattice constant L_p being fixed up to the elasticity effects from gravitation. While this lattice constitutes a preferred frame, the observable excitations obey Lorentz invariance due to the following four principles:

-The continuum limit (non-resolvability): Since the lattice is unresolvable for wavelengths $\lambda \gg L_p$, the discrete differences between lattice sites smooth out into continuous fields. The underlying absolute grid disappears from the equations of motion, replaced by a smooth spacetime manifold where only the relative coordinates of the wave-packets matter.

-Universal limiting velocity: The 'speed of light' in the model is not an independent constant but an emergent property of the medium. It is determined by the ratio of the elasticity of the Tetron interactions and their density. Since all excitations propagate through the same medium, they all share the same maximum propagation speed. This universal speed limit is the foundation of the Lorentz transformation.

-Hyperbolic dispersion relations: The excitations in the monolayer are not classical particles but collective modes which obey Lorentz-invariant wave equations like the Klein-Gordon or Dirac equation. Due to the restoration force of the ground state, these modes follow a relativistic dispersion relation: $E^2 = (pc)^2 + (mc^2)^2$.

-The internal nature of the observer: Since the observers (and their measuring devices) are themselves made of these same excitations, they are locked into the same wave dynamics. An observer moving through the monolayer cannot detect the static isospin cells because their own clocks and rulers change in unison. This is the Principle of Relativity realized within a medium.

4 The isomagnetic Interactions among Tetrons

How to obtain the above described ground state from a dynamics? It all starts with some unknown fundamental 7+1D interaction among tetrons, and then one has to assume that this induces exchange integrals which in turn lead to Heisenberg and DMI interactions among the isospin vectors, which finally are responsible for the tetrahedral order.

When trying to adapt these isospin interactions to the observed quark and lepton spectrum, a dominance of DMI over Heisenberg shows up[11, 12, 4], with the DMI Hamiltonian

$$\begin{aligned} H_{DMI} &= -D_{LL} \sum_{i \neq j=1}^{4} (\vec{Q}_{Li} \times \vec{Q}_{Lj})^2 - D_{LR} \sum_{i \neq j=1}^{4} (\vec{Q}_{Li} \times \vec{Q}_{Rj})^2 \\ &\quad - D_{RR} \sum_{i \neq j=1}^{4} (\vec{Q}_{Ri} \times \vec{Q}_{Rj})^2 \end{aligned} \tag{5}$$

Actually one should distinguish the cases for inter- and intra-cell interactions, so all of the couplings D appear in a D_{inter} and a D_{intra} version, both always being of the same order of magnitude. The result of an extensive analysis[4] shows, that the top mass is essentially determined by $D_{LL,intra}$, and the Fermi scale and the masses of the weak bosons by $D_{LL,intra}$. $D_{RR,intra}$ mainly measures the mass of the b-quark and $D_{LR,intra}$ of the τ-lepton. As explained below, there are also Heisenberg interactions in the system with much smaller couplings ≤ 1GeV and responsible for the masses of the second family.

Note that one has here a 'quadratic' type of DMI enforced by applying Moriya's rules[13] to the tetrahedral isospin configuration, in contrast to the 'linear DMI' form $\vec{D}_{ij}(\vec{Q}_i \times \vec{Q}_j)$ encountered in other situations[15, 16]. While the linear DMI is a chiral interaction, the quadratic DMI effectively is $\sim (\vec{Q}_i \vec{Q}_j)^2 - (\vec{Q}_i)^2 (\vec{Q}_j)^2$ and thus non-chiral, which means there is no apriori preference of the all-out configuration (which is left-handed) over the right-handed all-in configuration from the side of the Hamiltonian, but the choice of the all-out over the all-in configuration must be due to a sponteneous symmetry breaking[4, 6].

Another important point is that the sums exclude $i = j$, i.e. interactions of the Tetron wave function with itself are not allowed. This is important, because otherwise the interaction $\sim (\vec{Q}_{Li} \times \vec{Q}_{Ri})^2$, which drives $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ away from each other, would prevent the alignment of the $\vec{Q}_L$ and $\vec{Q}_R$ isospin structures.

As quantified in [4], the masses of the third family roughly correspond to the values of the DMI couplings

$$D_{LL} \approx m_t \qquad D_{LR} \approx m_\tau \qquad D_{RR} \approx m_b \tag{6}$$

As mentioned above, there is also a small Heisenberg Hamiltonian active on the system

$$H_H = -J_{LL} \sum_{i\neq j=1}^{4} \vec{Q}_{Li}\vec{Q}_{Lj} - J_{LR} \sum_{i\neq j=1}^{4} \vec{Q}_{Li}\vec{Q}_{Rj} - J_{RR} \sum_{i\neq j=1}^{4} \vec{Q}_{Ri}\vec{Q}_{Rj} \tag{7}$$

(again in an inter- and an intra-version). The Heisenberg couplings turn out to be responsible for the masses of the second family with

$$J_{LL} \approx m_c \qquad J_{LR} \approx m_\mu \qquad J_{RR} \approx m_s \tag{8}$$

and are therefore necessarily much smaller than the DMI couplings. One may then say that the tetrahedral ground state is a DMI-dominated isomagnet with a small admixture of Heisenberg couplings. (The relevance of the Heisenberg couplings for quark confinement will become clear later in the section about flux tubes.)

While a classic Heisenberg-dominated magnet favors symmetric, collinear alignment of spins, a quadratic DMI-dominated lattice penalizes collinearity and forces the vacuum into a highly frustrated, non-collinear all-out tetrahedral configuration rather than a flat, ordered state. Because the quadratic DMI $(\vec{Q}_i \times \vec{Q}_j)^2$ is 100 times stronger than the Heisenberg term, the system's ground state is deeply trapped in a steep and narrow potential well in configuration space, effectively freezing the vacuum and making any transverse fluctuations out of this tetrahedral alignment energetically unfavorable. Mathematically, this dominant quadratic DMI acts as a symmetric operator that concentrates the massive O(100) GeV energy scale essentially onto a single eigenvalue (the top mass), providing a natural origin for the heavy third-generation particles. Conversely, the lighter families remain protected and mass-free in this limit, escaping the crushing DMI scale because the quadratic interaction avoids the entangling degeneracies of standard linear DMI.

Note that each corner of the tetrahedron has 3 neighboring edges. If $(\vec{Q}_i \times \vec{Q}_j)^2$ is maximized (i.e. H_{DMI} minimized), every isospin tries to stand perpendicular to its 3 neighbors. The only configuation in the 3D spaces defined by Q_L and Q_R, that simultaneously optimzes this 'perpendicularity' condition, for all corners, is the all-out tetrahedral ground state. So actually, D_{LL} is the driving force of the SSB in the system and stabilizes the 2 aligned tetrahedrons T_L and T_R within the unit cell, as well as the inter-cell alignment, more than any Heisenberg interaction could ever do.

In other words, the secret of the DMI alignment is, that it does not – like the Heisenberg interaction – favor parallel or anti-parallel configurations, but maximal orthogonality in all components. Since the DMI is quadratic (proportional to the sine squared of the angle between the 2 vectors), the restoring force is even larger than for the linear DMI making the ground state isospin tetrahedrons extremely rigid structures.

5 Tetronic Origin of the Strong Interaction

A priori, to the microscopic model, QCD seems more of a foreign body. As we shall see, however, it may be derived as a compelling emergent extension, both local (at high energies) and non-local (at low energies) in nature and – at low energies – involving many unit cells, which confine any triplet(=quark) excitation. In essence this is an alternative, discrete approach to the observed strong forces, including the phenomena of asymptotic freedom and confinement, the latter having never been proven rigorously within the framework of continuous QCD. In fact, it is well known, and the proof will be done below, that a phenomenon like confinement is much easier to prove in a discrete environment like the Tetron monolayer than in the continuum.

Consider the 24 possible excitations of one unit cell. There are 6 singlet states (the leptons) and 6 triplets (corresponding to the 6 quark flavors), all transforming under the tetrahedral Shubnikov group G_4[4, 6]. The 3 wave funtions of a triplet can be understood as arising from symmetry functions adapted to the 4 tetrahedral corners as combinations $++--$, $+-+-$ and $+--+$, and actually can be traced back to the 3 rotations by 180° corresponding to the permutations 2143, 3412 and 4321 of the tertahedral/permutation group A_4[1] , while the singlets(=leptons) in this scheme correspond to $++++$.

More in detail, the contribution to the up-type mass eigenstates from one corner I

[1]The three π rotations together with the unit element form the Klein four-group V_4, which is a normal subgroup of the alternating group. The quotient group A_4/V_4 is isomorphic to the cyclic group Z3.

(and analogously for the corners II, III and IV) of the tetrahedron is

$$\begin{aligned}
u_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_1^2}} & [(|Q_{LIx}\rangle + \epsilon_1 |Q_{RIx}\rangle) + (|Q_{LIy}\rangle + \epsilon_1 |Q_{RIxy}\rangle) \\
& & +(|Q_{LIz}\rangle + \epsilon_1 |Q_{RIz}\rangle)] \\
c_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_2^2}} & [(|Q_{LIx}\rangle + \epsilon_2 |Q_{RIx}\rangle) + \omega(|Q_{LIy}\rangle + \epsilon_2 |Q_{RIy}\rangle) \\
& & +\bar{\omega}(|Q_{LIz}\rangle + \epsilon_2 |Q_{RIz}\rangle)] \\
t_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_3^2}} & [(|Q_{LIx}\rangle + \epsilon_3 |Q_{RIx}\rangle) + \bar{\omega}(|Q_{LIy}\rangle + \epsilon_3 |Q_{RIy}\rangle) \\
& & +\omega(|Q_{LIz}\rangle + \epsilon_3 |Q_{RIz}\rangle)]
\end{aligned} \tag{9}$$

and for the down quarks

$$\begin{aligned}
d_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_1^2}} & [(|Q_{RIx}\rangle - \epsilon_1 |Q_{LIx}\rangle) + (|Q_{RIy}\rangle - \epsilon_1 |Q_{LIy}\rangle) \\
& & +(|Q_{RIz}\rangle - \epsilon_1 |Q_{LIz}\rangle)] \\
s_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_2^2}} & [(|Q_{RIx}\rangle - \epsilon_2 |Q_{LIx}\rangle) + \omega(|Q_{RIy}\rangle - \epsilon_2 |Q_{LIy}\rangle) \\
& & +\bar{\omega}(|Q_{RIz}\rangle - \epsilon_2 |Q_{LIz}\rangle)] \\
b_I &= \tfrac{1}{\sqrt{3}\sqrt{1+\epsilon_3^2}} & [(|Q_{RIx}\rangle - \epsilon_3 |Q_{LIx}\rangle) + \bar{\omega}(|Q_{RIy}\rangle - \epsilon_3 |Q_{LIy}\rangle) \\
& & +\omega(|Q_{RIz}\rangle - \epsilon_3 |Q_{LIz}\rangle)]
\end{aligned} \tag{10}$$

where ω and $\bar{\omega}$ are the 120° phases of the Z_3 subgroup of G_4 given by $\omega = e^{i\frac{2\pi}{3}}$ and $\bar{\omega} = \omega^2$.

Three coefficients $\epsilon_{1,2,3}$ appear in these equations, which depend on the quark and even on the lepton masses. They can be calculated within the model[4] to be

$$\epsilon_i = \frac{1}{6}\frac{M_{Li}}{M_{Ui} + M_{Di}} \tag{11}$$

where M_{Ui}, M_{Di} and M_{Li} denote the corresponding masses within family $i = 1, 2, 3$.

These equations only reveal the weak isopin and family structure of the triplet excitation, based on the $Q_L \leftrightarrow Q_R$ and the $Z_3 \subset G_4$ symmetries of the unit cell, respectively. In order to distinguish the Shubnikov 'color' states one must include all 4 corners I, II, III and IV of the tetrahedron to form the symmetry adapted functions

$$
\begin{aligned}
|q_{++--}\rangle &= \frac{1}{\sqrt{4}}[q_I + q_{II} - q_{III} - q_{IV}] \\
|q_{+--+}\rangle &= \frac{1}{\sqrt{4}}[q_I - q_{II} - q_{III} + q_{IV}] \\
|q_{+-+-}\rangle &= \frac{1}{\sqrt{4}}[q_I - q_{II} + q_{III} - q_{IV}]
\end{aligned}
\tag{12}
$$

for any $q = u, c, t, d, s, b$ of the expressions (9) and (10).

According to (5)ff our tetrahedral system is dominated by the quadratic DMI, and the energy of the vacuum is minimized when the all-out condition is met. If one locally excites the system by a singlet or a triplet, one creates a local deviation from this vacuum. In particular, the triplet excitations create a divergence of isomagnetic charges across the tetrahedron. By flipping two corners relative to the other two, like in the excitation $++--$, there is a conflict with the global all-out vacuum, which must be compensated for by the neighboring cells. When a triplet alters the ground state across many cells, it leaves a track in the medium. A second triplet – note this must be an antiparticle wrt the base space – can then travel along this track with less energy expenditure. This represents the classical definition of a long-range attraction at low energy. It corresponds to a strong feedback topological effect.

So when a single triplet enters, it is a deformation that cannot just terminate abruptly. It spreads out extensively. Only when 2 triplets combine to a singlet (a meson), their isomagnetic charges add up to zero, corresponding to a G_4 symmetric state. In the near field between the two triplets, the field remains strongly distorted, whereas in the far field (outside the meson), the crystal returns to a perfect, undisturbed all-out state. The triplet pair effectively encapsulates the distortion.

Note, at this point this is just the qualitative idea. The details, how the 'color flux' is defined within the model, and squeezed into flux tubes will be described in the section about flux tubes.

Much in contrast, the singlet modes $(++++)$ represent a uniform scaling or a global rotation of all four isospins in the tetrahedron, in the sense that every isospin vector is moved in exactly the same way relative to the all-out 'axis'. Since the movement is symmetric across all four corners of the tetrahedron (rotational symmetry of the singlets), the total isomagnetic flux generated by this excitation sums to zero within the unit cell. As a result, a singlet does not create a source or sink of flux.

Building on the state definitions in Eq. (12), we can define the color-electric charge of a quark within the isomagnetic lattice. The color charge is not a continuous $SU(3)$ generator, but rather the topological asymmetry of the four internal vertices. While the macroscopic vacuum state as well as the lepton sector exhibit a fully symmetric, unfrustrated configuration $(++++)$, each of the three quark states represents a 180° rotation (a flip) or, equivalently, an internal sign permutation with two positive and two negative vertices. When embedded into the vacuum, the internal asymmetry of such a the state $|q\rangle$ necessarily induces a frustration at the cell boundary. It is precisely this topological boundary divergence that defines the color charge, as it forces the cell to emit a compensating, directed chain of non-trivial link operators (gluons) forming the color-electric flux tube. This will be worked out in detail in the section about flux tubes.

6 QCD Gluons Emerging from the Model

8 Shubnikov 'gluons' may be identified for the system. At first sight they are different than the $SU(3)$ gluons of QCD, but also non-abelian in nature, because moving e.g. from a $++--$ state to a $+-+-$ state involves a rotation that doesn't commute with other tetrahedral rotations. In G_4 they decompose as $1+1+3+3$, with a vector triplet 3 carrying the 'magnetic' flux, and an axial triplet providing currents that wrap around the flux and, as shown in the section about flux tubes, confine it.

In this section I will explain, how QCD gluons arise as gauge fields from the discrete geometric intuition. As noted before, at this point color is just the quantum number that distinguishes the 3 states within triplets of the G_4 Shubnikov group, and the gluons are identified as the transition operators that transform one state of a triplet into another. Mathematically, such an operator is a 3 times 3 matrix

$$\hat{G}_{\alpha\beta} = |q_\alpha\rangle\langle\bar{q}_\beta| - \frac{1}{3}\delta_{\alpha\beta}\sum_\gamma |q_\gamma\rangle\langle\bar{q}_\gamma| \tag{13}$$

acting on the 3-dimensional space spanned by the quark states (12)

$$\alpha, \beta \in \{++--, +-+-, +--+\} \tag{14}$$

How does the bilinear form $\hat{G}_{\alpha\beta}$ look geometrically between two of the tetrahedral unit cells? A quark state is a very specific, collective isospin twist (a phase texture) of the Tetron isospins. The bilinear form, on the other hand, corresponds to a dynamic oscillation or a tunneling process in which the twist of one cell on axis β propagates to another cell while tilting to axis α.

On this level, the bilinear forms represent 8 Shubnikov dof, and as stated, the Shubnikov 8 representation is not the same as the adjoint 8 of the continuous Lie group SU(3). However, one must distinguish the symmetry G_4 of the bare lattice from the symmetry of the emergent dynamics in the continuum limit of the discrete amorphous spatial Planck structure, cf. Section 3.

To understand what I mean, consider a real cubic crystal in 3D space. It only possesses the discrete symmetry of a cubic group O_h. A cube has no continuous rotational symmetry. It cannot be rotated by arbitrary angles without altering the lattice. Yet, if we consider the sound waves in this crystal, whose wavelength is much larger than the lattice spacing, the sound wave only sees a continuous, elastic medium, and its equations of motion follow the emergent symmetry of SO(3), which arises from smoothing over many unit cells.

Similarly here: if one could microscopically resolve the unit cell, one would see that the 8 vibrational modes do not have perfect SU(3) symmetry, but split into different symmetry classes according to $8 = 1 + 1 + 3 + 3$. The fact that we measure the perfect, irreducible 8 of SU(3) in the laboratory where all 8 gluons possess exactly the same strong coupling, is an illusion of the continuum limit.

Why, in contrast to sound waves, do we arrive at SU(3) instead of SO(3)? The answer is that the discrete quark states $|q_\alpha\rangle$ span a basis of a three-dimensional complex vector space ($\mathbb{C}^3$), because the isospins of the Tetrons are mathematically defined via complex spinors. As the lattice undergoes macroscopic, continuous deformation due to the propagation of the quarks, this effective $\mathbb{C}^3$ space of isospin deformations transforms accordingly. The mathematical group that describes unitary i.e. probability-preserving transformations in a complex $\mathbb{C}^3$ space is precisely $SU(3)$.

It must be stressed, that the deformations we are talking about do not concern the spatial 3D or 3+4D discrete structure but is purely a distortion of the internal isospin-tetrahedrons relative to one another.

7 The Gluon Field as a Gauge Field

The vacuum of the model is determined by the isospin couplings J and D. If one introduces a quark into one of the unit cells, this excitation forces a distortion of the surrounding all-out configuration, so that any arriving vibrational mode is scattered and phase-shifted. We want to relate the local gradient distortion of the crystal to a gauge potential, and its rigidity to the strong coupling constant. To that end we start with the matrix $\hat{G}$ defined in (13).

In the unperturbed vacuum, $\hat{G}$ formed by the Shubnikov quark states has a homogeneous reference value, which we define as $\hat{G}^{(0)}$. Since the modes are orthonormal (or can be orthonormalized), both $\hat{G}$ and $\hat{G}^{(0)}$ are unitary matrices (up to normalization). The local $SU(3)$ gauge element $g(\vec{r}, t)$ is then the unitary transition matrix that rotates the unperturbed reference modes of the global vacuum into the actually distorted local modes of the cell at position $\vec{r}$:

$$\hat{G}(\vec{r}, t) = g(\vec{r}, t) \cdot \hat{G}^{(0)} \quad \Longleftrightarrow \quad g(\vec{r}, t) = \hat{G}(\vec{r}, t) \cdot \left(\hat{G}^{(0)}\right)^{-1} \tag{15}$$

Any thermal or quantum mechanical fluctuation that locally tilts or twists the isospin vectors $\vec{Q}_a(\vec{r}, t)$ of the cell alters the wave functions $\hat{G}(\vec{r}, t)$ and thus directly modulates the local field $g(\vec{r}, t)$.

It may be noted, that in the continuous limit $g(\vec{r}, t) \in SU(3)_C$ describes the local gauge transformation, whereas at the level of the Tetron lattice it is nothing other than the local orientation matrix of the isospin tetrahedron relative to the global vacuum background.

One can now use the apparatus of standard gauge theory and define the gauge potential via a Maurer-Cartan form. Geometrically, in the Tetron model this means, if the orientation of the tetrahedra changes in time or spatially from one lattice cell to the next, a gradient distortion (shear/torsion) of the isospin vacuum arises. Mathematically, this is described by the derivative of g and leads to the gauge potential

$$A_\mu = -\frac{i}{g_s} g^\dagger \partial_\mu g = -\frac{i}{g_s} \hat{G}^\dagger \partial_\mu \hat{G} \tag{16}$$

If one goes back to the definition of $\hat{G}$ in (13), one sees that the gluon field can be interpreted as the generalized non-Abelian Berry connection (or geometric phase) of the Shubnikov quark states[17, 18]. When a quark state propagates through the isospin lattice, the gluon field it experiences is not an independent, fundamental force. It is the purely geometric phase resulting from the inner Shubnikov states $|q_\alpha(r,t)\rangle$ slightly rotating from one lattice site to another in spacetime. The 'color charge' is simply the information indicating which of these three orthogonal states the particle currently occupies.

Concerning the time component, the term $g^{-1}\partial_t g$ describes the local angular velocity (or the quantum mechanical phase velocity) with which the isospin vectors in the cell $\vec{r}$ oscillate around their equilibrium position. In classical mechanics, this corresponds to gyroscopic motion. A_0^a then is the projection of this local precession velocity onto the Gell-Mann matrices.

Spacetime covariance of A_μ is not an artificial requirement here, but results directly from the fact that the waves of isospin distortions propagate as collective modes in the monolayer with the effective 'speed of sound' of the amorphous lattice, which in the continuous limit takes on the role of the speed of light c.

If one writes down the equation of motion for an elastic isomagnetic wave in such an already pre-deformed system, the ordinary derivative automatically transforms into a covariant derivative

$$D_{\mu,\alpha\beta} = \partial_\mu \delta_{\alpha\beta} + i g_s A_\mu^a \lambda_{\alpha\beta}^a \tag{17}$$

The gluon fluctuations transfer momentum between the quarks because any movement of a quark compresses or stretches the vibrational modes of the surrounding Tetron crystal. The system permanently attempts to minimize the elastic energy of the Heisenberg and DMI couplings, which macroscopically manifests on the monolayer as the observed $SU(3)$ color force.

8 Flavor Independence

When describing the gluon in terms of 2-cell $q\bar{q}$ excitation states like in (13), one must actually be cautious to guarantee flavor independence. A rigorous lattice description must ensure the flavor universality of the strong interaction: the eight $SU(3)_c$ gluons must mediate identical interactions for all six quark flavors (u, d, s, c, b, t); i.e. the gluon should be understood topologically as the universal process of color transfer across any inter-cell boundary Σ, only dependent on the V_4 transition, but independent of which Z_3 state (9) or (10) one choses.

When 2 colored fermions at cell A and cell B combine, the gluon wave function $\Psi_g^a(A, B)$ emerges as the overlap integral

$$\Psi_g^a(A, B) \approx \int_\Sigma d^4\xi \, [\bar{\chi}_A(\xi) \cdot \lambda^a \cdot \chi_B(\xi)] \tag{18}$$

Here, $\chi(\xi)$ represents the internal cellular quark wavefunction, and λ^a are the eight Gell-Mann generators of $SU(3)_c$.

The SU(2) representation spaces, on which the isospin tetrahedrons live, are 3-dimensional, and therefore 2 adjacent unit cells meet at a shared 2D interface. (The boundary between two 3D volumes is mathematically a 2D surface.) Crucially, the transition dynamics across this interface are not determined by the internal flavor state of the quark (such as its generational excitation mode), but exclusively by the fundamental lattice couplings of the vacuum at the boundary – specifically the symmetric exchange couplings J_{inter} and the antisymmetric interactions D_{inter} defined after (5). The color operators λ^a strictly commute with both the Z_3 flavor rotations and the $SU(2)_{L/R}$ isospin operators[4]. Just as the boundary couplings J_{inter} and D_{inter} they depend solely on the geometry of the tetrahedral interface.

To make this more explicit one should consider Eq. (12) which shows that the property to be a Shubnikov quark triplet has nothing to do with the $Z_3 \subset G_4$ symmetry in (9) (describing the family structure) nor with pseudospin transitions $T_L \leftrightarrow T_R$ (corresponding to weak isospin (9)$\leftrightarrow$(10)) but solely refers to the three 180°-rotations of the tetrahedral Shubnikov group G_4 which connect the states (12). When a gluon is formed at the wall between 2 cells A and B (as a 2-cell excitation), only this property gets encoded into the gluon.

The critical distinction lies in how these topological properties project across the 2D inter-cell boundary. When the gauge fields are evaluated via the overlap integral of the wavefunctions at the shared interface, the boundary color operator couples exclusively to the color polarization inversions dictated by the three 180°-rotations of the tetrahedral unit cell. Because this acts as a normal subgroup – the famous Klein four-group V_4 of A_4 – its operations strictly commute with both the cyclic Z_3 generation phases $(1, \omega, \bar{\omega})$ and the $L \leftrightarrow R$ amplitudes in (9) and (10). Consequently, while the W boson actively mediates an $L \leftrightarrow R$ transition that projects across Z_3 flavor axes[4], the gluon operator acts entirely within the decoupled color subspace of G_4. When the gluon transition amplitude is integrated across the wall, the internal Z_3 flavor phases and isospin parameters mathematically factor out, geometrically guaranteeing that the exact same fundamental mode of color flux is excited regardless of the quark's flavor.

9 Asymptotic Freedom

It is a well-known exercise to derive the standard QCD Lagrangian from an $SU(3)$ gauge structure with covariant derivative (17). Then, in ordinary continuous QCD, the β-function tracks how virtual particle loops hide the short-distance infinities of smooth space. Because the Tetron model is fundamentally discrete at the Planck scale, albeit amorphous wrt physical space, the continuous running of the coupling constant is replaced by an effective field theory description:
–At all experimentally accessible distances up to a length scale $L_E(\gg L_p)$, which lies between a PeV and the GUT scale, the random distribution of cells averages out perfectly. The vacuum looks like a smooth continuum. Here, one may use a standard effective β-function to describe how the coupling changes because one is averaging over trillions of cells. Only very much above the weak scale, at temperatures where entire regions of isomagnetic order begin to melt or a multi-phase mixture forms, this picture looses its meaning.
–Then, in between L_E and L_p, the concept of a running coupling constant breaks down. Even more, the ordered isospin vacuum ceases to exist, together with all physical particles (its excitations). Only the bare structural couplings of the discrete Tetron units remain. Even though the cells can fluctuate and are distributed randomly due to their elastic interactions, they cannot squeeze infinitely close together, because the direct Tetron-Tetron fermionic coupling remains like a hard-core repulsive potential on the Planck scale.

–Finally, approaching the Planck energy, the monolayer evaporates into a 7D Tetron gas.

Crucially, because the physical unit cells of the 3D base space remain localized up to L_E, the quarks behave as perfectly point-like Dirac particles for all experimentally accessible energy regimes, bypassing experimental constraints on spatial quark compositeness. Of course, one may ask whether effects e.g. in jet cross sections can arise from the tendency to 'melt' of the local tetrahedral isospin order even below L_E, when 2 high energy particles hit at TeV-energies.

However, this concern can be countered by considering that a single high-energy particle (even at 100 TeV) cannot simply melt a local tetrahedral unit cell, as this cell is topologically and energetically anchored within the infinite collective of its amorphously distributed neighboring cells.

10 The Strong CP Problem

In the Tetron model, the color flux (the gluons) at the 2D cell wall arises exclusively through the overlap of the V_4 states (12). These linear combinations are strictly real (the coefficients are merely $+1$ and -1).

For physical CP violation via the well-known $F\tilde{F}$ term, a complex phase in the strong sector is strictly required (often manifesting as an imaginary part in the determinant of the quark mass matrix, which couples to the gauge field via the chiral anomaly). However, because the operator structure defining the strong interactions at the wall is generated by the purely real 180° color polarization flips (± 1), imaginary components simply do not appear in the color space. The rigidity of the real Klein four-group V_4 geometrically forces the complex-valued topological phase term θ_{QCD} to be exactly zero, and so the appearance of the $F\tilde{F}$ term is forbidden in the microscopic model.

This may be contrasted to the situation in the weak sector. Whereas the strong interaction is based on the real V_4 symmetry, the weak interaction mixes the states of the Z_3 family symmetry. As seen in (9) and (10), the Z_3 symmetry contains the complex phases ω and $\bar{\omega}$ (with $\omega = e^{2\pi i/3}$). When the W boson projects across the tilted axes, it acquires these complex phases. As shown in [4], these complex Z_3 phases are the basis for the construction of the PMNS and CKM matrices and the associated CP-violation within the present model.

In conclusion, the Tetron model separates CP conservation in the strong interaction from CP violation in the weak interaction purely through the fundamentally different algebraic geometry (real 180° flips vs. complex 120° rotations) within the 4D isospin space.

11 Confinement and the Formation of Flux Tubes

As seen in the above sections, the naked gauge invariant QCD Lagrangian can be proven to arise in the continuum limit of the Tetron theory. It remains to reflect, whether the inherent discreteness of the system modifies the understanding of the low energy behavior of quarks and gluons. As discussed on the following pages, the mechanism of confinement is somewhat different than in the Mandelstam interpretation of low energy QCD. This alternative interpretation is not in contradiction to the QCD Lagrangian (which describes the local kinematics of the gauge fields and is entirely dictated by symmetry) but describes a different non-perturbative global vacuum state.

The basic idea is that the quark confinement scale is determined by the inter-cell Heisenberg couplings J_{inter} which enforce the color exchange against the rigid tetrahedral DMI lattice. As will be proven below by an explicit calculation, the much larger inter-cell DMI couplings $D_{inter} = O(100)$ GeV do not affect the string tension relevant for confinement[2] because the DMI is blind to the formation of the tube.

When one pulls quarks apart, one is not overpowering the massive 100 GeV DMI frame; one is merely stretching the softer 1 GeV Heisenberg springs that connect the Isospins within that frame. This is why the strong interaction dynamics manifests at the 1 GeV scale, even though the structural integrity of the vacuum itself is safely anchored at 100 GeV by the DMI.

[2]Due to the phenomenon of induced ordering[19] the relevant scale is $D_{LL,inter}$ and not $D_{RR,inter}$ or $D_{LR,inter}$.

Before I go into the details of these considerations I want to compare 3 appraches to Meissner-type formation of flux tubes:

(i) Inside a standard type II superconductor a condensate of charged particles (Cooper pairs) expels magnetic flux (Meissner effect), so that magnetic flux tubes (Abrikosov-Vortizes) are formed. These flux tubes do *not* connect electric charge, but they guide and enclose a quantized magnetic field ($\Phi_0 = h/2e$), while superconducting circular currents (the Cooper pairs) enclose and circulate around the non-superconducting core of a flux tube. An interesting point is that if magnetic monopoles would exist and put in a Type-II superconductor they would be connected by Abrikosov-Vortizes. So the magnetic field lines between them do not disperse throughout space like in vacuum, but are squeezed into the extremely thin, thread-like channel of the the flux tube. Because the energy of the flux tube grows linearly with its length, a constant, attractive force arises between the north and south monopole that does not decrease even at infinite distance. You can never separate the two monopoles from each other (confinement). Note, Type-II superconductivity occurs almost exclusively on a solid-state substrate with a regular lattice structure, because it is based on phonons attracting 2 electrons to form a Cooper pair.

(ii) Let me compare this to the standard picture of QCD confinement. Since the QCD Lagrangian views quarks as 'electric' color charges, an inverted model was proposed by Mandelstam and others[1, 20] to explain confinement: the 'dual' Meissner Effect. There it is assumed that the vacuum is a condensate of magnetic monopoles (instead of Cooper pairs) and that this condensate expels color-electric fields, so that color-electric flux tubes are formed, and that these connect the quark's color-electric charges (instead of the magnetic monopoles) and lead to the linear confinement of quarks observed in the strong interactions. Note that in contrast to superconductors, the QCD vacuum may be modeled as an amorphous, fluid-like condensate of magnetic monopoles. Magnetic monopoles are assumed to condense in the vacuum just like electrons form Cooper pairs in a standard superconductor. Unlike a crystalline solid, however, this quantum vacuum is completely disordered, isotropic, and fluid-like, without any fixed lattice structure ('amorphous'). The big drawback of the Mandelstam picture is the rather artificial assumption of the existence of a monopole vacuum.

(iii) In the present model, confinement occurs through a dual Meissner effect, just as in (ii). The 'knack', however, is that the tetrahedral all-out system is already a vacuum condensate of oriented Q_L-Q_R isospin pairs (isomagnetic 'Cooper pairs'). These pairs represent local phase locking at the 2D intercell walls, keeping the all-out isospin tetrahedrons globally aligned in parallel. The idea for confinement is then that since the vacuum disfavors any isomagnetic flux that contradicts the all-out order, it does not let the isomagnetic flux lines spread out on the monolayer, but squeezes them into a narrow flux tube. The quarks in the Tetron model are not inserted foreign particles, but isomagnetic vibrations corresponding to tiny tetrahedral misalignments, which prevent the vacuum from dispersing the color lines in all directions. Similar to the monopoles in the Mandelstam QCD vacuum fluid picture, the unit cells are distributed amorphously on physical space.

	Condensate/Vacuum	**Confined Charges**
(i) Superconductor	Electric Cooper Pairs	Magnetic Monopole
(ii) Mandelstam QCD	Color Magnetic Monopoles	Color Electric Quarks
(iii) Tetron Model	Isomagnetic $Q_L Q_R$ Pairs	Triplet G_4 excitations

Table 1: Comparison between type-II superconductivity, the Mandelstam QCD vacuum, and the Tetron model.

In the following I shall take some time to detail the approach described in (iii). Starting point of the discussion will be the underlying Tetron dynamics, with its strict energetic hierarchy between the macroscopic chiral ordering and the local inter-cell exchange interaction ($D_{inter} \gg J_{inter}$).

Because the chiral $Q_L - Q_R$ alignment is stabilized by the dominant scale D_{inter}, the tetrahedral lattice structure is so rigid that the vacuum condensate does not 'evaporate' or melt inside the flux tube. Instead, the extreme rigidity acts as the primary confinement mechanism, by forbidding the color flux from spreading isotropically into the 3D monolayer. Because deforming the D_{inter} order is energetically prohibited, the vacuum is forced to compress the color field into a one-dimensional channel.

While D_{inter} dictates the tubular geometry of the confined flux, the actual energy density (the QCD string tension of 0.3 GeV/fm) is governed by the softer Heisenberg scales J_{inter}. At the 2D boundary between adjacent cells, the quantum mechanical overlap of the highly directional isospin vectors is naturally described by the Heisenberg exchange interaction $J_{inter}(\vec{Q}_A \cdot \vec{Q}_B)$. When a quark and an antiquark are separated, they induce a discrete 180° topological flip in the V_4 Shubnikov states (12) along the connecting path. This color-flip abruptly inverts the sign of the local geometric overlap across the cell boundary, generating a sharp, localized energy penalty dictated entirely by J_{inter}.

Therefore, the phase inside the flux tube is a color-frustrated vacuum. The flux tube consists of a sequence of physically stable unit cells whose internal V_4 color orientation is topologically misaligned relative to the surrounding transverse vacuum. The macroscopic string tension observed in strong interactions is the accumulated geometric 'friction' – the integral of the discrete Heisenberg exchange penalties – generated across the lateral cell walls where the frustrated Shubnikov states interface with the unperturbed, rigid lattice.

A single quark (=triplet excitation) without an isospin partner would generate a shear stress in the tetrahedral order that cannot simply vanish into nothing. Instead, the elastic distortion of the field propagates far into the crystal. Since the stiffness D_{inter} is extremely high, this long-range perturbation costs a massive amount of energy, about 100 GeV. Normally, it would send its isospin flux lines out in all directions. However, since the DMI implies a massive energy penalty, the vacuum pushes back on the flux lines from every side. The flux lines find that they can minimize the damage by staying as close together as possible. Instead of spreading into all 3 base space dimensions, the flux is squeezed into a 1D line (a tube) connecting the triplet to an anti-triplet.

The quantitative proof of those statements will be given in the next section. For the moment, I will turn to the singlet modes (leptons l_{++++}), which escape the fate of confinement, because they represent a uniform scaling or a global rotation of all four isospins in the tetrahedron. Since the movement is symmetric across all 2×4 corners in the ground state of the unit cell, the total isospin flux generated by this excitation sums to zero. So there is no need for a singlet to be connected to another excitation by a flux tube. It can travel through the monolayer as a solitary wave. The energy cost is purely local. To move to the next cell, it just has to transfer its local deformation. This leads to a standard dispersion $E(k)$ rather than a linear potential $V(r) \propto r$.

12 J_{inter} and the String Tension

To establish a rigorous mathematical foundation for the confinement mechanism within the Tetron lattice, we must analyze the exact energetic contributions on the microscopic level, i.e. at the boundary between adjacent unit cells. A defining feature of the model is the vast hierarchy between the macroscopic vacuum rigidity scale ($D_{inter} \approx 100$ GeV) and the Heisenberg couplings ($J_{inter} \approx J_{LL,LR,RR} < 1$ GeV). Note once again that we are assuming throughout this paper that intra- and inter-cell couplings have the same order of magnitude, cf. the remark after (5).

Now we demonstrate that the quadratic DMI (5) and the Heisenberg exchange (7) couple to fundamentally different topological degrees of freedom. This asymmetry guarantees that the DMI scale acts solely as a geometric confiner of the color flux tube thickness, while the linear string tension is dictated exclusively by the Heisenberg scale.

Let us define the discrete boundary between Cell A and Cell B, coupled at their four corresponding vertices $\alpha \in \{I, II, III, IV\}$ as in (12). The isospin vectors at these vertices are denoted as $\vec{Q}_A^{(\alpha)}$ and $\vec{Q}_B^{(\alpha)}$. We analyze two transitions:
(i) the Vacuum-to-Vacuum transition. In this case Cell A and Cell B are both in the unfrustrated all-out vacuum state abbreviated as $++++$. Thus, $\vec{Q}_B^{(\alpha)} = +\vec{Q}_A^{(\alpha)} = \langle\vec{Q}^\alpha\rangle$ for all α, where the angle brackets denote the vacuum expecation value.
(ii) the vacuum-to-quark transition. In this case Cell A is in the vacuum state $(++++)$, while Cell B contains a frustrated Shubnikov color state (e.g., $++--$). Thus, two vertices are perfectly parallel, and two are perfectly anti-parallel: $\vec{Q}_B^{(\alpha)} = \pm\vec{Q}_A^{(\alpha)}$. More precisely, $\vec{Q}_A^{(\alpha)} = \langle\vec{Q}^\alpha\rangle$ as before, but $\vec{Q}_B^{(\alpha)} = \delta\vec{Q}^\alpha = q_\alpha$ in the notation of (12).

To identify the string tension as originating from the inter-cell Heisenberg interaction, we compare the transition energy with the DMI case. The Heisenberg Hamiltonian is given in terms of the scalar product promoting parallel alignment of the isospin vectors in cell A and cell B

$$H^J_{\text{inter}} = -J_{inter} \sum_{\alpha=1}^{4} \vec{Q}^{(\alpha)}_A \cdot \vec{Q}^{(\alpha)}_B \tag{19}$$

Evaluating this for the vacuum-to-vacuum transition yields the ground state energy

$$E^J_{\text{vac}\to\text{vac}} = -J_{inter} \sum_{\alpha=1}^{4} \vec{Q}^{(\alpha)}_A \cdot (+\vec{Q}^{(\alpha)}_A) = -4J_{inter} \tag{20}$$

In contrast, for the vacuum-to-quark transition, where two vertices flip their sign, the scalar products sum to zero

$$E^J_{\text{vac}\to\text{quark}} = -J_{inter}\,(1+1-1-1) = 0 \tag{21}$$

The transition from a trivial vacuum link to a topological color flux link thus incurs a strict energetic penalty of

$$\Delta E^J = E^J_{\text{vac}\to\text{quark}} - E^J_{\text{vac}\to\text{vac}} = +4J_{inter} \tag{22}$$

Because any macroscopic flux tube connecting a quark-antiquark pair over a distance R requires a discrete Wilson line of such 180° cell-wall flips, this fixed energy penalty per cell wall directly generates the linear confinement potential $V(R) = \sigma R$. The string tension σ is thus governed entirely by J_{inter}.

Boundary Transition	$\mathbf{\Delta E^{\text{DMI}}}$ **(Rigidity)**	$\mathbf{\Delta E^{J}}$ **(String Tension)**
Vacuum → Vacuum	0	0
Vacuum → Quark	0	$+4J_{\text{inter}}$

Table 2: Relative energetic penalties at the unit cell boundary. The unfrustrated vacuum link is defined as the zero-energy baseline. The D_{inter} term is completely blind to the discrete 180° color flip, leaving the J_{inter} term as the sole generator of the linear confinement potential.

Now for the quadratic DMI Term, which is responsible for the rigid potential walls of the isospin lattice. This term penalizes non-collinear alignments via the square of the cross product

$$H^{\text{DMI}}_{\text{inter}} = D_{\text{inter}} \sum_{\alpha=1}^{4} \left(\vec{Q}^{(\alpha)}_A \times \vec{Q}^{(\alpha)}_B\right)^2 \tag{23}$$

Evaluating this term for the pure vacuum transition (0° alignment) yields a trivial zero

$$E^{\text{DMI}}_{\text{vac}\to\text{vac}} = D_{inter} \sum_{\alpha=1}^{4} \left(\vec{Q}_A^{(\alpha)} \times (+\vec{Q}_A^{(\alpha)}) \right)^2 = 0 \tag{24}$$

Crucially, evaluating this Hamiltonian for the topological color flip (180° anti-alignment for two vertices) also yields exactly zero, as the cross product of any vector with its exact opposite vanishes:

$$E^{\text{DMI}}_{\text{vac}\to\text{quark}} = D_{inter} \sum_{\alpha=1}^{4} \left(\vec{Q}_A^{(\alpha)} \times (\pm\vec{Q}_A^{(\alpha)}) \right)^2 = 0 + 0 + 0 + 0 = 0$$

So we conclude $\Delta E^{\text{DMI}} = 0$ which means there is no energetic penalty from the DMI term on the formation of the color flux tube.

In conclusion, the vacuum-to-vacuum transition is energetically preferred wrt to vacuum-to-quark by $4J_{\text{inter}}$, while for the DMI there is no energetic difference between the 2 transitions. While the Heisenberg term is sensitive to discrete 180° rotations ($=V_4$ sign permutations), the quadratic DMI is blind to them. Since $D_{\text{inter}} \gg J_{\text{inter}}$ the DMI acts as an immense energetic funnel. Because the penalty scales as $D_{\text{inter}} \sin^2(\theta)$, it violently suppresses any continuous, soft spatial twisting of the isospin field ($\theta \neq 0, \pi$). It forces the lattice dynamics to remain collinear. Consequently, to avoid the large D_{inter} penalty, topological color flux cannot propagate via smooth continuous deformations. The lattice is rigidly forced to route the flux through the only loophole available: the discrete 180° flips. The fact that this happens in a 1D-channel and does not spread out radially into the monolayer, has a simple geometric reason: a 1D-chain of N frustrated unit cells necessarily contains less internal walls than a 2D- or 3D-Cluster of the same number of such cells, and thus the total energy (penalty) of the flux configuration given by $N \times 4J_{inter}$ is smaller for a 1D-chain.

To summarize, within this enforced 1D channel, the energetic cost of the flux is handed over entirely to the Heisenberg scalar product, correctly producing a string tension somewhat below the 1 GeV scale of J_{inter}. This understood, it may be asked, what the role of the DMI coupling is wrt confinement. The answer is that the large value of the DMI coupling $D_{inter} \approx 100 \times J_{inter}$ fixes the rigidity of the vacuum configuration, so that it plays the role of a 'guardian' of the system, because any transverse spreading of the color-electric flux would induce a quadratic penalty from the vacuum rigidity $\propto D_{inter} \sin^2(\theta)$. Consequently, the flux is forced to propagate exclusively via discrete 180° phase flips, with energy $N \times 4J_{inter}$ given above.

If we explain quark confinement this way one may ask what is the connection to the way gluons were defined as 2-cell excitations in (13). Our system has a little resemblance with lattice gauge theory, and in the language of LGT the question can be answered by saying that quarks live on the sites (the cell volumes) because a quark is a frustrated single isospin cell (e.g., $++--$), while gluons live on the links (the 2D cell walls). As bilinears, gluons act as transition operators between cells. A gluon (13) contains the information how the color state changes as one crosses the wall from Cell A (α) to Cell B (β). The frustrated wall energy we have calculated above (vacuum to quark, or traversing the flux tube) carries a color mismatch. This mismatch is the $\bar{q}_\alpha q_\beta$ bilinear excitation.

13 Conclusions

In this paper, we have demonstrated that Quantum Chromodynamics and its phenomena can be derived as emergent macroscopic properties of a 7+1-dimensional Tetron model. By analyzing the topological and geometric properties of the discrete tetrahedral vacuum, we have bridged the gap between the discrete lattice dynamics of fundamental Dirac Tetrons and the continuous effective field theory of the strong interaction.

In this approach, the local $SU(3)_C$ color gauge symmetry is not postulated as an a priori physical input, but rather emerges naturally from the internal degrees of freedom of the composite quark states. Within the Tetron framework, quarks are formulated as excitations of $SU(2)_L \times SU(2)_R$ tetrahedrons, while gluons emerge from a topological transport as the localized phase transitions of excitations hopping between adjacent tetrahedral cells. In the continuum limit ($L_p \to 0$), the discrete phase factors accumulate during this transport as the standard gauge connections A^a_μ of a $SU(3)$ gauge theory.

The model provides a unique, UV-complete physical mechanism for both low-energy confinement and high-energy asymptotic freedom. We have shown that the Heisenberg coupling $J_{inter} \leq 1$ GeV, which governs the scalar 'hopping' energy between next-neighbor cells, is physically identified with the strong interaction scale Λ_{QCD} and with the string tension of the color-electric flux tubes which have been explicitly constructed within the model.

While the standard continuous QCD Lagrangian is successfully recovered in the continuum limit due to the universality of the dynamically generated $SU(3)_C$ gauge symmetry, the Tetron model departs from the standard Mandelstam picture in its microscopic IR mechanism. Rather than postulating a phenomenological condensate of fundamental magnetic monopoles, the dual Meissner effect in the Tetron framework makes use of the already existing isomagnetic Q_L-Q_R pairing. The further analysis has shown that it is a direct consequence of the geometric frustration and chiral boundary conditions of the all-out isospin tetrahedral structure.

So the confining medium here is the rigid, frustrated geometry of the isospin lattice itself which is formed by isomagnetic condensates. This vacuum strictly expels color-electric fields, forcing the emergent gauge flux into discrete, one-dimensional channels. In the macroscopic limit, these channels smooth out into isotropic flux tubes, yielding the characteristic linear potential $V(r) \propto r$ and a rigorous proof of color confinement.

In summary, QCD is the effective, low-energy macroscopic limit of topological Tetron dynamics. The strong force, color charge, and confinement are no longer viewed as arbitrary, postulated properties of elementary particles. Instead, they are revealed to be the inevitable consequences of a tetrahedral vacuum structure in $SO(4)$ isospin space. This framework opens up new pathways for calculating non-perturbative QCD observables directly from the discrete topological invariants of the tetrahedral lattice.

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