MPI-PhT/98-10

NLO QCD Corrections and Triple Gauge Boson Vertices at the NLC

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Abstract

We study NLO QCD corrections as relevant to hadronic W decay in W pair production at a future 500 GeV e^+e^- linac, with particular emphasis on the determination of triple gauge boson vertices. We find that hard gluon bremstrahlung may mimic signatures of anomalous triple gauge boson vertices in certain distributions. The size of these effects can strongly depend on the polarisation of the initial e^+e^- beams.

Although the Standard Model is in excellent agreement with existing collider data, there are strong grounds to expect discrepancies to appear once future high energies accelerators are commissioned. In particular, Physics Beyond the Standard Model may show up in the form of anomalous triple gauge boson vertices. Although LEP data will constrain these trilinear couplings, high statistics at energies far above threshold are needed to pinpoint small deviations from the Standard Model. Thus detailed analyses have been performed on the sensitivity of W pair production and decay at future high energy e^+e^- linacs [1] [2] [3] to the presence of anomalous triple gauge boson vertices. These studies, however, have not taken into account NLO QCD effects in hadronic W decay (see however [4]). In this letter, we will argue that these effects can generate deviations from tree level Standard Model predictions which are large enough to influence discovery bounds on anomalous form factors.

The differential cross-section for W pair production and decay (in the narrow width approximation which we use throughout following [5],[1]) can be schematically written as

$$d\sigma \sim \frac{\Gamma_a \Gamma_b}{\Gamma^2} \sum_{\lambda \tau_1 \tau_2 \tau_1' \tau_2'} F^{\lambda}_{\tau_1 \tau_2}(s, \cos\vartheta) F^{*\lambda}_{\tau_1' \tau_2'}(s, \cos\vartheta) D_{\tau_1 \tau_2} D_{\tau_1' \tau_2'} \tag{1}$$

where F is a generic helicity amplitude dependent on $\sqrt{s}$ and production angle ϑ, and D_{ab} denotes a generic element of the density matrix for W decay. λ denotes the electron helicity ($\pm\frac{1}{2}$) while τ denotes $W^\pm$ helicities (+,−,0). Γ is the total width of the W while Γ_a and Γ_b denote partial widths to the final states of interest. The precise forms of F and D may be found in [5]. The constants we have not explicitly written in Eq. 1 are purely kinematical overall factors common to all final states and which are not relevant for the numerical results we will present later on.

For the sake of convenience, we reproduce the diagonal elements of D from [5]

$$D_{++} = \tfrac{1}{2}(1+\cos^2\theta) \quad -\{\cos\theta\} \tag{2}$$
$$D_{--} = \tfrac{1}{2}(1+\cos^2\theta) \quad +\{\cos\theta\} \tag{3}$$
$$D_{00} = \sin^2\theta \tag{4}$$

where θ is the polar angle of the outgoing fermion in the rest frame of the decaying W and all fermions are assumed massless. We will assume that the rest frame of each W can be reconstructed, thereby excluding purely leptonic final states. For hadronic W decays, where the jet charges cannot be reconstructed, symmetrisation between quarks and anti-quarks requires that the terms within $\{\}$ must be dropped. The off diagonal elements of D depend, in addition to θ, on the azimuthal angle ϕ, but make no contribution to the total cross-section. They are however relevant for azimuthal correllations, which we will not discuss.

Once NLO QCD effects are included in W decays, the formulae above must be modified. In addition to θ and ϕ, the matrix elements for gluon bremstrahlung contributions depend on additional phase space variables, and are singular in the collinear limit, quite apart from divergences due to virtual corrections. As these singularities cancel only when IR safe quantities are calculated, IR safe generalisations of θ and ϕ are required. One way of proceeding follows from the observation that θ in Eq. 2, defined in terms of the quark direction, is at LO also the polar angle of the thrust direction. As thrust is IR safe [6], NLO D functions defined in terms of the thrust orientation, which for our purposes is the direction of the most energetic outgoing parton in the W rest frame, are gaurenteed to be singularity free.

Restricting ourselves once again to diagonal matrix elements we have at NLO [7]

$$D_{AA} = (1+\frac{\alpha_s}{\pi})(1-3LC_F\frac{\alpha_s}{2\pi}) \times \{D^0_{AA} + 2LC_F\frac{\alpha_s}{2\pi}\} \tag{5}$$

neglecting terms of $\mathcal{O}(\alpha_s^2)$. D^0 denotes the symmetrised leading order term and L is a numerical constant of value .4875 which is a relic of numerical integration over Dalitz variables. All angles in Eq. 5 now refer not to a given outgoing parton but to the thrust axis. As before, all outgoing partons are assumed massless. It should be noted that there are additional terms linear in $\cos\theta$ in Eq. 5 which vanish once we assume that jet charges are not determined, and which we have therefore dropped.

Retaining terms to $\mathcal{O}(\alpha_s)$ only

$$\sum_A D_{AA} = (1+\frac{\alpha_s}{\pi})\sum_A D^0_{AA}$$

The term in brackets is the well known $\mathcal{O}(\alpha_s)$ QCD K factor for hadronic W decay, as expected. We thus see, that for observables for which polarisation is not relevant, the NLO corrections may be obtained by rescaling LO results by a constant factor which we will derive shortly. In the Standard Model where gauge cancellations ensure the suppression of longitudinal gauge boson production at high energies, polarisation may be relevant for some observables, for example asymmetries, where different polarisation states are in general weighted differently. Hence we will not only approximate NLO effects by a constant K factor, but will also make use of Eq. 5 convoluted with Eq. 1.

Before proceeding further along these lines, it is useful to study the possible significance of NLO QCD effects in the analysis of anomalous gauge boson vertices. To do so, it is instructive to consider the term within {} in Eq. 5 for $A = +$. This term can be rewritten as

$$\frac{(1+\cos^2\theta)}{2}(1+2LC_F\frac{\alpha_s}{2\pi}) + \frac{\sin^2\theta}{2}2LC_F\frac{\alpha_s}{2\pi}$$

For transversely polarised gauge bosons, the LO distribution gets rescaled and in addition a longitudinally polarised component seems to appear. However, the appearance of additional longitudinal modes is one of the hallmarks of non-standard triple gauge boson vertices ! Thus we see that NLO QCD has the potential to mimic signatures of anomalous triple gauge boson vertices. The size of this effect is proportional to L, indicating that hard gluon bremstrahlung is responsible. It is significant that in Eq. 5 α_s is evaluated at M_W independent of $\sqrt{s}$; NLO QCD corrections thus do not diminish in size with increasing energy, in contrast to other radiative corrections such as initial state bremstrahlung and finite width contributions.

For longitudinal modes ($A = 0$) it is easy to see from Eq. 2 and Eq. 5 that the QCD K factor at $\theta = 0$ is infinite. Thus large QCD corrections may be expected in distributions where longitudinal modes are important. The infinite K factor is due to the vanishing of the LO cross-section at $\theta = 0$, which may be understood in terms of angular momentum conservation, and has been observed in other processes involving hadronic W decay [8].

To derive expected K factors, it is important to note that NLO QCD effects also appear in the redefinition of the width and branching fractions of the W, which appear in Eq. 1. As we will focus on final states containing leptons and hadrons, what appears at Born level is (in obvious notation), $\frac{\Gamma_L\Gamma_H}{\Gamma^2}$ where all widths are calculated from tree level expressions. Keeping terms up to and including $\mathcal{O}(\alpha_s)$ only [1]

$$\Gamma^2 \rightarrow \Gamma^2(1+2\frac{2}{3}\frac{\alpha_s}{\pi})$$

The NLO cross-section with no phase space cuts can be obtained from Born level by making a further change *i.e.*

$$\Gamma_H \rightarrow (1+\frac{\alpha_s}{\pi})\Gamma_H$$

Thus the change to the cross-section can be accounted for by rescaling by a factor $\overline{K}$ given by

$$\overline{K} = (1-\frac{1}{3}\frac{\alpha_s}{\pi}) \tag{6}$$

neglecting terms with higher powers of α_s. This is the constant factor mentioned earlier.

As we have argued earlier, the sum total of NLO QCD corrections may not always be obtained by rescaling by the constant factor in Eq. 6. Therefore we will define a C, a modified K factor as follows;

$$C = \frac{(NLO - \overline{K}LO)}{\overline{K}LO} \tag{7}$$

If C vanishes or is very small for a certain observable in a given region of phase space, then a rescaling is sufficient to describe NLO effects. If this is not the case, then NLO effects are significant. Similarly, we can define C_A, to describe the corrections due to anomalous couplings as follows,

$$C_A = \frac{(A - LO)}{LO} \tag{8}$$

where A is the contribution for a certain choice of anomalous triple gauge boson vertices.

We will now present numerical results for various observables at LO and NLO to illustrate the size of corrections to be expected. We fix throughout, $\sqrt{s} = 500\ GeV$, $M_Z = 91.187\ GeV$, $M_W = 80.33\ GeV$, $\alpha = \frac{1}{128}$, $\sin\theta_W{}^2 = .23$, and $\alpha_s = .12$. For the sake of definiteness, we assume that the W^- decays hadronically, and the W^+ leptonically. The incoming e^+e^- beams are

[1]This factor is set to 1 in [7] due to a different choice of overall normalisation.

unpolarised unless otherwise stated, and the anomalous contributions are defined via Eq. 8. Angles referring to outgoing fermions and jets are defined in the rest frame of the parent $W^{\pm}$.

To begin with, we demonstrate the utility of absorbing $\overline{K}$ into the LO cross-section by studying the C dependence of the differential cross-section with respect to θ_-, the polar angle of the thrust axis. The two curves in Fig. 1 correspond to C with $\overline{K}$ defined by Eq. 6 and $\overline{K} = 1$. We see immediately that in this instance a significant part of the NLO corrections can be absorbed into the redefinition of widths, thereby reducing the magnitude of such effects.

However, this is not always the case. For example, for observables for which the LO contributions vanish in certain regions of phase space, NLO effects cannot be simply accounted for by a redefinition of widths. One such observable discussed in [7] is given by

$$\int_{-1}^{1} d\cos\theta_+ \frac{\cos\theta_+}{|\cos\theta_+|} \frac{d\sigma(e^+e^- \to \ell^+\nu j_- X)}{d\cos\vartheta d\cos\theta_- d\cos\theta_+} \tag{9}$$

which corresponds to the double differential distribution with respect to $\cos\vartheta$ and $\cos\theta_-$, with the azimuthal angles integrated over and the polar angle of the charged anti-lepton integrated over anti-symmetrically.

This observable may seem rather contrived, however, being asymmetric by construction it is sensitive to the C and P Violating form factor denoted by z_z in [5]. As can be seen from Table 1, NLO QCD effects and a non zero value of z_z both generate appreciable corrections to the Standard Model predictions for the distribution described in Eq. 9, particularly for $\cos\vartheta \sim 0$. The need for taking into account NLO QCD effects in the analysis of anomalous triple gauge boson vertices is apparent.

As has been pointed out in [5] and [1], triple differential distributions are particularly sensitive to anomalous gauge boson couplings. Hence, as a further illustration of the relevance of Eqs. 5 we now sample a triple differential distribution with all azimuthal angles integrated over and the polar angle of the thrust axis is fixed at 0.1. In addition, the incoming beams are polarised (e^-_R and e^+_L). The non-zero anomalous couplings are choosen to be (in the notation of [5])

$$x_\gamma = .005 \quad \delta_z = \frac{x_\gamma}{\sin\theta_W \cos\theta_W} \quad x_z = -x_\gamma \frac{\sin\theta_W}{\cos\theta_W}$$

This choice of parameters is motivated by the scenario in [9] and the values above are slightly above the threshold for discovery at $\sqrt{s} = 500\, GeV$ according to LO analyses in [1].

¿From the results in Table 2 it is clear that although the anomalous couplings produce sizable deviations from the tree level predictions of the Standard Model, NLO QCD effects are definitely not negligible in comparision and need to be taken into account to establish discovery limits.

It is worth noting that for opposite incoming beam helicities, both the anomalous corrections and NLO QCD corrections in the same region of phase space are much smaller; the NLO QCD corrections are never more than a few percent. This can be understood from the fact that for incoming e^-_R, the outgoing W^- is largely longitudinally polarised, while for incoming e^-_L the outgoing W^- is largely transverse. This strong dependence of the size of NLO QCD effects on the incoming beam polarisation has not been pointed out before, and is particularly significant, as several authors have suggested beam polarisation as a diagnostic tool to unravel the structure of anomalous gauge boson interactions [2] [10]. It is also noteworthy that Tables 1 & 2 are so different from each other, indicating that NLO QCD corrections to different observables may not be simply obtained from some universal prescription, but must be calculated from scratch.

Common to the results presented in Tables 1 and 2, apart from the strong dependence of K factors with phase space, is the fact that different polarisation states make different contributions to the distributions under consideration, either due to a choice of initial polarisation or due to an observable being asymmetric by construction. As we have argued earlier it is precisely in such cases that NLO QCD corrections could be non-trivial and this is indeed consistent with our results, and with Figs. 1-3 of [7] where a sizable variation in K factors is also observed. This suggests a useful rule of thumb; higher order QCD corrections should not be approximated by constant K factors where W polarisation is observed and/or where different polarisation states make different contributions to the observables under consideration. Finally, it is worth repeating that the magnitude of the relevant K factors is controlled by $\alpha_s(M_W)$, and is thus independent of $\sqrt{s}$ insofar as the narrow width approximation is valid. It is not surprising therefore, that the broad features of Tables 1 and 2 persist at higher energies as well.

To summarise, we have demonstrated the importance of NLO QCD corrections in the analysis of triple gauge boson vertices at future e^+e^- linacs.

The magnitude of these corrections varies strongly with beam polarisation and seem to be particularly large for asymmetries, and certainly will affect the exclusion bounds for anomalous triple gauge boson vertices. A precise quantitative estimate will be possible only with a detailed analysis including detector acceptances which is beyond the scope of this letter, but is definitely worth undertaking.

Acknowledgements KJA wishes to thank Jose Wudka for valuable clarifications and encouragement. BL acknowledges useful discussions with J.G. Körner.

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Captions

Figure 1

The variable C with $\overline{K}$ defined in Eq. 6 (dashed line) and $\overline{K} = 1$ (solid line) are plotted as a function of θ_-, the polar angle of the thrust axis, with all other angles integrated over. θ_- is retricted by $0 < \theta_- < \frac{\pi}{2}$ as jet charge is assumed not to be identified.

Table 1

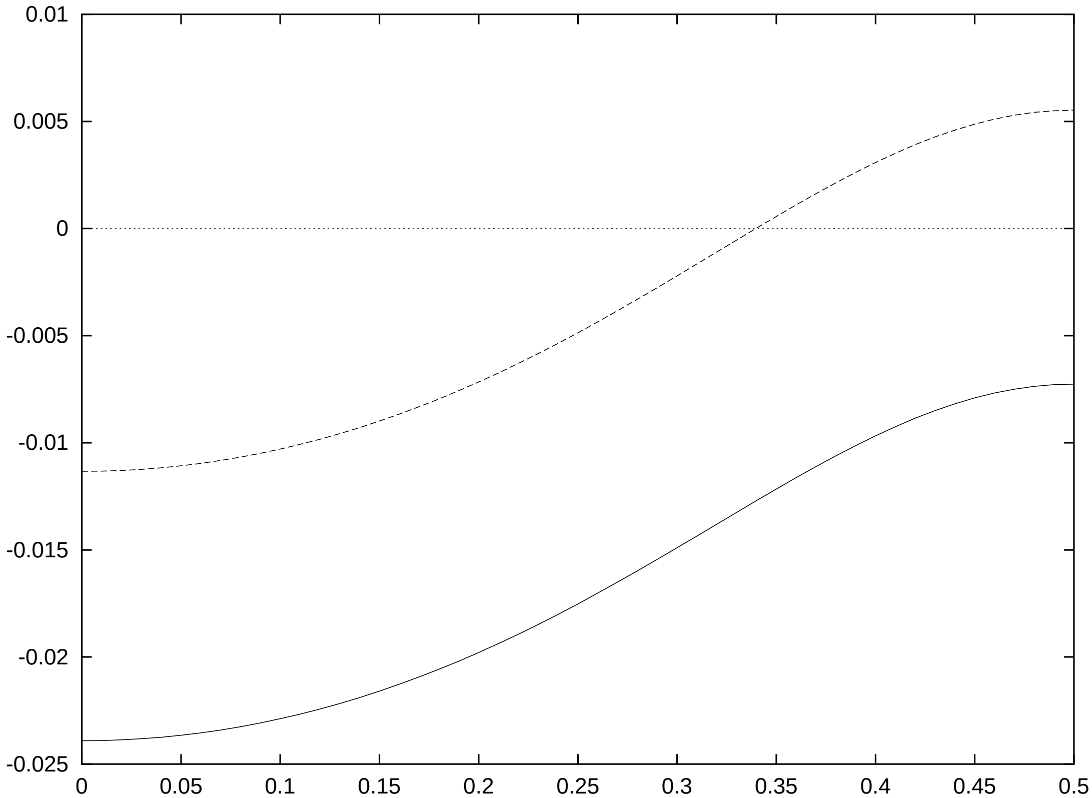
The values of C and C_A are plotted to three significant figures as a function of $\cos\vartheta$ and θ_- for the variable defined in Eq. 9;the upper figure in each entry is C, defined the text, while the lower figure is C_A evaluated for $z_z = .001$, with all other anomalous parameters set to 0. $\cos\vartheta$ runs along the vertical axis and θ_- (in units of π) along the horizontal axis.

Table 2

The values of C and C_A are plotted as a function of $\cos\vartheta$ and θ_+ for the double differential cross-section evaluated at $\theta_- = .1$ with polarised beams (e^-_R and e^+_L). The upper figure in each entry is C, while the lower figure is C_A evaluated for anomalous parameters described in the text. $\cos\vartheta$ runs along the vertical axis and θ_+ (in units of π) along the horizontal axis. All azimuthal angles have been integrated over.

Table 1

	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
-.8	-.012 .000	-.012 .001	-.011 .003	-.010 .007	-.008 .012	-.005 .019	-.002 .027	.001 .036	.004 .044	.006 .050	.007 .052
-.7	-.014 .000	-.014 .000	-.013 .002	-.011 .004	-.009 .007	-.006 .011	-.002 .016	.002 .022	.006 .027	.008 .032	.010 .033
-.6	-.015 .000	-.015 .000	-.014 .001	-.012 .003	-.010 .005	-.006 .008	-.002 .011	.002 .015	.007 .019	.010 .022	.011 .023
-.5	-.016 .000	-.015 .000	-.014 .001	-.013 .002	-.010 .003	-.007 .006	-.002 .008	.003 .012	.008 .015	.011 .017	.013 .018
-.4	-.016 .000	-.016 .000	-.015 .001	-.013 .002	-.011 .003	-.007 .005	-.002 .007	.003 .010	.009 .013	.013 .016	.015 .017
-.3	-.017 .000	-.017 .000	-.016 .001	-.014 .002	-.011 .003	-.007 .005	-.002 .008	.004 .011	.010 .015	.015 .018	.017 .019
-.2	-.019 .000	-.018 .000	-.017 .001	-.015 .002	-.012 .004	-.008 .007	-.003 .011	.005 .017	.013 .022	.020 .028	.023 .030
-.1	-.023 .000	-.022 .000	-.021 .002	-.019 .005	-.016 .009	-.011 .016	-.003 .028	.010 .045	.028 .071	.050 .101	.061 .117
0.	∞	.973 -.061	.220 -.061	.081 -.061	.033 -.061	.010 -.061	-.001 -.061	-.008 -.061	-.012 -.061	-.014 -.061	-.014 -.061
.1	-.007 .000	-.007 -.001	-.006 -.002	-.006 -.004	-.005 -.008	-.003 -.011	-.002 -.015	-.001 -.019	.000 -.022	.001 -.025	.001 -.025
.2	-.011 .000	-.011 .000	-.010 -.001	-.009 -.002	-.007 -.004	-.005 -.006	-.002 -.009	.000 -.012	.003 -.014	.004 -.016	.005 -.016
.3	-.012 .000	-.012 .000	-.011 -.001	-.010 -.002	-.008 -.003	-.005 -.004	-.002 -.006	.001 -.008	.004 -.010	.006 -.012	.007 -.012
.4	-.013 .000	-.013 .000	-.012 -.001	-.010 -.001	-.008 -.002	-.006 -.003	-.002 -.005	.001 -.006	.005 -.008	.007 -.009	.008 -.009
.5	-.014 .000	-.013 .000	-.012 .000	-.011 -.001	-.009 -.002	-.006 -.003	-.002 -.004	.002 -.005	.005 -.006	.008 -.007	.009 -.007
.6	-.014 .000	-.014 .000	-.013 .000	-.011 -.001	-.009 -.001	-.006 -.002	-.002 -.003	.002 -.004	.005 -.005	.008 -.005	.009 -.005
.7	-.014 .000	-.014 .000	-.013 .000	-.011 .000	-.009 -.001	-.006 -.001	-.002 -.002	.002 -.003	.006 -.003	.009 -.004	.010 -.004
.8	-.014 .000	-.014 .000	-.013 .000	-.011 .000	-.009 .000	-.006 -.001	-.002 -.001	.002 -.002	.006 -.002	.009 -.002	.010 -.002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0.1
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0.3
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0.4
0.45
0.5

Figure 1:

Table 2

	0	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
-.8	1.551	.874	.393	.218	.144	.106	.085	.072	.064	.058	.054
	.290	.252	.225	.216	.212	.210	.209	.208	.208	.207	.207
-.7	1.275	.783	.389	.234	.167	.133	.113	.101	.093	.087	.084
	.314	.269	.233	.219	.212	.209	.208	.207	.206	.206	.206
-.6	1.062	.701	.382	.249	.189	.158	.140	.130	.122	.117	.114
	.333	.284	.240	.221	.213	.209	.207	.205	.205	.204	.204
-.5	.892	.627	.373	.261	.209	.182	.167	.157	.151	.146	.143
	.348	.296	.246	.224	.214	.209	.206	.204	.203	.203	.203
-.4	.753	.559	.360	.269	.226	.204	.191	.182	.177	.173	.171
	.360	.307	.252	.226	.215	.209	.205	.203	.202	.201	.201
-.3	.638	.496	.345	.273	.239	.221	.210	.204	.199	.197	.194
	.371	.315	.256	.229	.215	.208	.204	.202	.201	.200	.200
-.2	.541	.439	.327	.273	.247	.233	.225	.220	.217	.214	.213
	.379	.322	.260	.230	.216	.208	.204	.201	.200	.199	.199
-.1	.458	.385	.306	.267	.248	.238	.233	.229	.227	.226	.225
	.387	.328	.263	.231	.216	.208	.204	.201	.199	.199	.198
0.	.386	.336	.282	.256	.243	.237	.233	.231	.230	.229	.229
	.393	.331	.264	.232	.216	.208	.203	.201	.199	.198	.198
.1	.323	.290	.256	.240	.232	.228	.226	.225	.224	.224	.225
	.399	.333	.264	.231	.216	.208	.204	.201	.199	.199	.198
.2	.268	.248	.227	.218	.214	.212	.212	.211	.211	.212	.213
	.403	.333	.263	.231	.216	.208	.204	.201	.200	.199	.199
.3	.219	.208	.198	.193	.192	.191	.191	.191	.192	.193	.194
	.408	.331	.260	.229	.215	.208	.204	.202	.201	.200	.200
.4	.175	.170	.167	.165	.165	.165	.166	.166	.167	.169	.171
	.412	.326	.256	.227	.215	.209	.205	.203	.202	.201	.201
.5	.135	.135	.135	.135	.136	.136	.137	.138	.139	.141	.143
	.415	.319	.250	.225	.214	.209	.206	.204	.203	.203	.203
.6	.100	.102	.103	.104	.105	.106	.107	.108	.109	.111	.114
	.418	.309	.244	.222	.213	.209	.207	.205	.205	.204	.204
.7	.067	.071	.072	.073	.074	.075	.076	.077	.079	.081	.084
	.421	.294	.236	.219	.213	.209	.208	.207	.206	.206	.206
.8	.038	.041	.042	.043	.043	.044	.045	.047	.048	.051	.054
	.424	.274	.228	.216	.212	.210	.209	.208	.208	.207	.207

Table 2 *cont.*

	.5	.55	.6	.65	.7	.75	.80	.85	.90	.95	1.
-.8	.054	.051	.048	.047	.045	.044	.043	.043	.042	.041	.038
	.207	.207	.208	.208	.209	.210	.212	.216	.228	.274	.424
-.7	.084	.081	.079	.077	.076	.075	.074	.073	.072	.071	.067
	.206	.206	.206	.207	.208	.209	.213	.219	.236	.294	.421
-.6	.114	.111	.109	.108	.107	.106	.105	.104	.103	.102	.100
	.204	.204	.205	.205	.207	.209	.213	.222	.244	.309	.418
-.5	.143	.141	.139	.138	.137	.136	.136	.135	.135	.135	.135
	.203	.203	.203	.204	.206	.209	.214	.225	.250	.319	.415
-.4	.171	.169	.167	.166	.166	.165	.165	.165	.167	.170	.175
	.201	.201	.202	.203	.205	.209	.215	.227	.256	.326	.412
-.3	.194	.193	.192	.191	.191	.191	.192	.193	.198	.208	.219
	.200	.200	.201	.202	.204	.208	.215	.229	.260	.331	.408
-.2	.213	.212	.211	.211	.212	.212	.214	.218	.227	.248	.268
	.199	.199	.200	.201	.204	.208	.216	.231	.263	.333	.403
-.1	.225	.224	.224	.225	.226	.228	.232	.240	.256	.290	.323
	.198	.199	.199	.201	.204	.208	.216	.231	.264	.333	.399
0.	.229	.229	.230	.231	.233	.237	.243	.256	.282	.336	.386
	.198	.198	.199	.201	.203	.208	.216	.232	.264	.331	.393
.1	.225	.226	.227	.229	.233	.238	.248	.267	.306	.385	.458
	.198	.199	.199	.201	.204	.208	.216	.231	.263	.328	.387
.2	.213	.214	.217	.220	.225	.233	.247	.273	.327	.439	.541
	.199	.199	.200	.201	.204	.208	.216	.230	.260	.322	.379
.3	.194	.197	.199	.204	.210	.221	.239	.273	.345	.496	.638
	.200	.200	.201	.202	.204	.208	.215	.229	.256	.315	.371
.4	.171	.173	.177	.182	.191	.204	.226	.269	.360	.559	.753
	.201	.201	.202	.203	.205	.209	.215	.226	.252	.307	.360
.5	.143	.146	.151	.157	.167	.182	.209	.261	.373	.627	.892
	.203	.203	.203	.204	.206	.209	.214	.224	.246	.296	.348
.6	.114	.117	.122	.130	.140	.158	.189	.249	.382	.701	1.062
	.204	.204	.205	.205	.207	.209	.213	.221	.240	.284	.333
.7	.084	.087	.093	.101	.113	.133	.167	.234	.389	.783	1.275
	.206	.206	.206	.207	.208	.209	.212	.219	.233	.269	.314
.8	.054	.058	.064	.072	.085	.106	.144	.218	.393	.874	1.551
	.207	.207	.208	.208	.209	.210	.212	.216	.225	.252	.290