

GENERAL RELATIVITY FROM ELASTICITY OF TETRON BONDS

Prerequisites

Needed for this project are the basics of the tetron idea, i.e. that the universe is an elastic medium built from 'tetron' constituents. The tetrans yield an internal tetrahedral symmetry constructed in such a way that the SM particle physics SSB and interactions and the family structure of quarks and leptons arise.

All known elementary particles are considered as quasiparticle wave-excitations propagating on the tetron medium.

Furthermore, familiarity with Einstein GR and the FLRW metric is assumed.

The FLRW universe in the light of the tetron model

For the case $p=0$ and $\Lambda=0$ Einstein's theory of gravity leads to the (acceleration form of the) Friedmann equation

$$d^2a/dt^2 = -4\pi/3 * G * \rho * a \quad (1)$$

It is independent of the curvature parameter k , i.e. independent of whether the universe is elliptic or hyperbolic.

Eq (1) looks pretty much like a linear harmonic Hooke-type acceleration for the cosmic expansion parameter a . The reason why one does not, however, encounter harmonic solutions has to do with the fact that the matter(=quasiparticle) density ρ is not constant but dilutes in the process of expansion according to the simple volume effect $\rho*a^3=const$.

Taking the dilution into account, one is led to the ordinary solutions of the Friedmann equation.

However, the dilution should not make one forget, that originally there are linear elastic forces between the tetron constituents (which built up the universe), and that the reaction of the elastic tetron material to energy-momentum follows simple linear Hooke-laws like (1).

Actually one can ascribe a Lamé constant to the universe, a bulk modulus

$$B=1/LP/LP/G$$

where LP is the Planck length and G Newton's constant.

In cms units B has the value

$$B=10^{112} \text{ kg/m/s/s}$$

a very large number corresponding to an extremely large stiffness of the tetron material, which in turn corresponds to a very high characteristic frequency ω_{Planck} of the material.

Cosmological Constant (CC) and Dark Energy (DE) in the Light of the Tetron Model

One can extend the presented analysis to include a cosmological constant Λ and matter with pressure p . Friedmann's acceleration equation then reads

$$d^2a/dt^2 = -4\pi/3 * G * (\rho+3*p) * a + \Lambda*a/3 \quad (2)$$

Note that linearity in a is maintained on the rhs and that for an empty universe one formally has solutions with sine/cosine (for $\Lambda < 0$) and exponential (for $\Lambda > 0$).

The case with $\Lambda > 0$ and the associated exponential growth of the universe is often suggested as a possible solution to the dark energy phenomenon. Writing the Einstein equations in the form

$$R(\mu,\nu) - R*g(\mu,\nu)/2 = 8\pi*G*T(\mu,\nu) - \Lambda*g(\mu,\nu) \quad (3)$$

one sees that one can understand the CC term as a vacuum energy contribution to the energy-momentum tensor. Since for a perfect fluid the energy-momentum tensor is given by

$$T(\mu,\nu) = (\rho+p)*u(\mu)*u(\nu) + p*g(\mu,\nu)$$

one can formally associate a negative pressure

$$p = -\rho = -\Lambda/8\pi/G \quad (4)$$

to a CC $\Lambda > 0$, i.e. a pressure which contributes to the further expansion of the universe, and just what is needed to account for the dark energy effect.

In the framework of the tetron model, where the universe is an elastic medium built from elastically bound microscopic constituents, a CC term with $\Lambda > 0$ is not really natural, because a medium with an exponential growth towards infinity sounds like rather strange stuff.

The most natural choice for a vacuum energy term in the framework of the tetron model is a linear Hooke-like elasticity force, where at large times the medium's expansion tends to a constant equilibrium value ($a = a_s$). As a consequence, the last term in eq. (2) should be modified according to

$$d^2a/dt^2 = -4\pi/3 * G * (\rho+3*p) * a - w^2 * (a_s - a) \quad (2')$$

where a_s is larger than the present extension of the universe, so that the last term in (2') has the same (positive) sign as the last term in (2).

The new vacuum energy term can be used to accommodate the observed dark energy effect provided one chooses $w = 10^{-18}$ Hz, corresponding to an extremely slowly varying oscillation towards a_s .

Formally, it can be thought of as arising from a contribution to the energy-momentum tensor (3) with a pressure $p = -\rho = +w^2*(1-a_s/a)/8\pi G$ replacing eq (4), i.e. instead of a pressure which expands the medium towards infinity the expansion tends towards a_s from below.

Physically, the new term corresponds to the linear spring force between the constituents of the elastic medium. In contrast to the CC term, which was introduced by Einstein without any intuition about the possible physical origin of the vacuum energy, in the tetron model it can be associated to the binding energy among the tetronic constituents of the elastic medium.

Its characteristic frequency w has to be distinguished from the effects of mass-energy on the medium, the latter with strength and characteristic time scales defined by Newton's constant G . While G measures the reaction of the elastic medium to any kind of mass-energy, w is determined by the intrinsic binding among the medium's constituents.

Reviewing the project's goal

We have used the FLRW metric to prove that the medium which makes up our universe shows linear elastic behavior. To arrive at the announced goal of the project it is necessary to extend the analysis to any possible metric, not just FLRW.

Starting such a program one immediately faces several questions:

-What happens to the nonlinear components of the Einstein action?

Preliminary Answer: they must probably be interpreted as non-linear contributions to the elastic interactions.

-Some of the curvature effects in Einstein theory are due to acceleration (=timely Curvature like d^2a/dt^2), others to spatial curvature. What is the difference from the standpoint of elasticity?

Answer: In timely or acceleration curvature the distance LP between the constituents of the elastic medium gets enhanced.

Spatial curvature, on the other hand, corresponds to buckling and bulging of 3dim physical space within the surrounding 6dim space.

To explain what is meant by the 'surrounding 6dim space' one should look at the global ground state of the tetron model. This model is designed not only to describe gravity effects but also particle physics phenomena like spontaneous symmetry breaking and parity violation of the weak interactions, and it is in this connection that the 'surrounding 6dim space' appears, cf the figure.

Global ground state of the universe

This figure shows the global ground state of the tetron model after the electroweak SSB. The big black arrow represents 3dim physical space, while the tetrahedrons extend into a 3dim internal(=isospin) space. Each tetrahedral point represents a tetron [=a fermion transforming as 8 under $SO(6,1)$], and the arrows indicate a tetron's isospin vector. The set of excitations of the isospin vectors behaves similar to magnons in solid state physics and can be interpreted as the spectrum of quarks and leptons.

Note that the isospin vector alignment is associated to the electroweak symmetry breaking, because at temperatures above the Fermi scale (before the SSB) the isospins in each tetrahedron are oriented randomly (not shown) and there is a corresponding local $SU(2)$ symmetry which gets broken when the isospin vectors align.

The figure also shows how the universe looks like in the tetron model. It is a 3-dimensional 'monolayer' of internal tetrahedrons whose average distances are given by the Planck length LP.

Gravity is due to the elasticity of the coordinate bonds between neighboring tetrahedrons and corresponds to tiny deviations from this average in the temporal, the vertical or the horizontal direction.

Digression on Big Bang and Inflation

In previous updates the behavior of the universe at late times was discussed, and it was shown that one can add a term $w^2(1-a_s/a)*g(\mu,\nu)$ to the energy momentum tensor in order to satisfy both the needs of the tetron model and the dark energy phenomenon. Assuming a potential $V=a/r^n-b/r^m$ (4) which near equilibrium takes the approximate form $V_0+V_1*(r-r_s)^2+\dots$ one is able to physically understand the values of the dark energy frequency w and the cosmological equilibrium scale factor a_s .

On the other hand it is clear that no cosmological model will be complete without understanding big bang and inflation, i.e. knowing what happens at early times.

At first sight it seems difficult to imagine an elastic medium whose existence starts at a point singularity. Actually, in the tetron model the big bang is replaced by a condensation process of tetrans where a large amount of energy is set free leading to a rapid expansion(=inflation) of the condensing medium.

More in detail, the following steps lead to the creation of our universe:

(i) The tetron point of view on the big bang actually starts before the big bang, assuming the existence of a cloud of tetrans in the original 6dim space which for energy reasons react to give a cloud of tetrahedral 'molecules', the building blocks of the universe formed later on.

(ii) it is then assumed that there is an attractive force among the tetrahedrons, so that their initial gaseous assembly gets compressed. This force is (not identical but) analogous to the gravity forces which leads to the compression of e.g. molecular clouds in interstellar space and to the formation of stars. Note, it is a force among tetrans in the original 6dim space, not among ordinary matter. In order that the whole system does not shrink to a point, in addition there should be repulsive forces between the tetrahedrons at very small distances like in eq. (4).

(iii) When the pressure in the gas of tetrahedral 'molecules' becomes larger and larger, at some point there is a condensation of the tetrahedrons leading to the ground state depicted in the figure shown in the last update. This is similar to what we know from ordinary matter which tends to condense to more solid forms when pressure is increased (provided temperature is not varied too much).

To understand this point consider the Gibbs free energy $dG=-SdT+Vdp$ which for constant temperature is $G(p_2)=G(p_1)+\int_{p_1}^{p_2} Vdp$. In other words, at higher pressure the side of the equation with less volume is energetically favored.

(iv) For reasons which have to do with the interactions among the tetrans, the condensation within 6dim space leads to a 3dim 'monolayer'/'carpet' of tetrahedrons, which is our 3dim physical universe.

Our universe has only one domain, because competing condensation processes lead to different 'carpets' which extend into different directions within 6dim space.

(v) Free tetrans do not exist in our universe because they are bound within the elastic medium with Planck energy. This energy is set free in the condensation and leads to an initial rapid expansion of the elastic medium, a process which is usually known as inflation.

Note that in contrast to the quasi-particle excitations (=ordinary matter) tetrans and thus the expanding metric are not bounded by the speed of light.

(v) The quasi-particles (=ordinary matter) as well as any kind of mass/energy slow down the expansion after the period of inflation.

This effect becomes the smaller the larger the universe, until a point is reached when re-accelerating starts towards a_s (= the dark energy effect described before).

Munich, June 2018

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